A characterization of Rawls's social welfare function via coherent risk measures

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Abstract

Relying on basic results about coherent risk measures in Banach spaces, we characterize, almost effortlessly, the Rawlsian welfare function when its domain is the set of bounded sequences. Our characterization theorem may be used to justify the use of Rawls's welfare function and also enables us to develop a formal framework underpinning the thought experiment (i.e., the "original position") originally envisioned by Rawls.

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1 Introduction

According to the ethical system put forward by Rawls (1971), the welfare of society should be measured on the basis of the welfare of the society's worst-off member. Rawls' own argument supporting the maximin welfare criterion rests on the presumption that people are "very" risk averse. Specifically, Rawls envisages the choice problem of a social welfare criterion in the original position as one under complete ignorance (i.e., a group of individuals are "behind the veil of ignorance" as they do not know yet what position they will occupy in the economy). So, assuming that individuals are "risk averse", he argues that a rational person would propose a social welfare function based on how they would view social states if they were to end up as society's worst-off member. In the case of infinitely-many agents, theorems giving an axiomatic characterization of Rawls' criterion of social welfare are already known, e.g. [Miyagishima (2015), Bosmans and Ooghe (2013)].

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In this paper we prove a result (Proposition 1) in the setting of infinite utility streams which characterizes the Rawlsian social welfare function in terms of "prudent" coherent risk measures. We use such a result to develop a mathematical and conceptual foundations of the Rawls's social welfare function, as follows: we study two different scenarios. In the first scenario (section 3), a social planner has to somehow choose among long-term policies (i.e., economic policies that affect the welfare of present and future generations) that give rise deterministically to a stream of utilities (where each utility value of the sequence represents the welfare of the respective generation); in this scenario, our Proposition 1 below (see section 4) puts the following intuition on a firm mathematical ground: a "prudent" social planner would choose the Rawls's social welfare function as the criterion guiding the selection of an optimal long-term economic policy.
In the second scenario (section 5), there are countably-many social states (or conceivable social positions like, e.g., homeless, unemployed, civil servant, teacher, engineer, billionaire etc.) and a social planner has to choose, ex-ante, a sequence of utilities $(u_1, u_2, u_3, \cdots)$, where each utility value represents the well-being of a person occupying the corresponding social position (i.e., if one lets social state 1 be "homeless" then u_1 is the well-being of someone who ends up being homeless, etc.). The question that arises in this scenario can be put as follows: what kind of social welfare function should the planner use to aggregate infinite sequences of utilities? Of course, once a welfare function has been appropriately chosen the society should select the utility sequences that maximize the value of that function.[1] We answer this question by building a mathematical framework (that combines the proof of Proposition 1 with well-known results about coherent risk measures in Banach spaces) which underpins Rawls's original thought experiment that, as mentioned above, was conceived of as an argument in favor of the maximin welfare criterion. More to the point, in section 5 of the paper we prove that if one were to put the aforementioned question to a selfish individual whose characteristics (i.e., available information and "attitude toward risk") reflect those that were examined originally by Rawls, proposing the Rawls's social welfare function (as Rawls himself argued) would be the rational (i.e., utility-maximizing) choice of such an individual. This is because on the one hand an individual equipped with maximin preferences can be regarded as a person with the above-mentioned characteristics, i.e., a person that "acts under a veil of ignorance" and is "afraid" to become the most disadvantaged member of society; on the other hand it turns out, remarkably, that the multi-prior expected utility function (that represents such preferences) is equal to the Rawls's social welfare function.

The remainder of the paper is organized as follows: in section 2, we remind the reader of the definition of coherent risk measure and we gather some known related results that will be exploited in this work; we also define the so-called prudence property that will be useful for making sense of the notion of prudent social planner. In section 3, we lay out formally the first scenario mentioned above. Section 4 is devoted to the proof of our characterization theorem and contains a discussion of the latter in relation to the construct of prudent social planner which is featured in the first scenario of section 3. In section 5, we formalize the aforementioned second scenario as a background underlying Rawls' original thought experiment. In section 6 we make some final remarks concerning the thrust of our contribution.

[1] Incidentally, note that one might think of the setting we have just described as a way to conceptualize and model the notion of acceptable levels of inequality in society.

2 Preliminaries: risk measures

In what follows, we put together some useful facts and concepts, about coherent risk measures, that will be used in the rest of the paper. They are taken and rearranged from Kountzakis and Polyrakis (2013) to which we refer the reader for a detailed account. We also use the background laid out in this secton to define the notion of prudent risk measure.

Let E be a Banach space ordered by the (non-trivial) cone P, and let $x_0 \in P$ be an order unit of E. A function $\rho : E \to \mathbb{R}$ is said to be a coherent risk measure if it satisfies the following properties:

1. $x \geqslant y$ implies $\rho(x) \leqslant \rho(y)$.
2. $\rho(x + tx_0) = \rho(x) - t$ for any $t \in \mathbb{R}$.
3. $\rho(x + y) \leqslant \rho(x) + \rho(y)$ for every $x, y \in E$.
4. $\rho(\lambda x) = \lambda \rho(x)$ for every $x \in E$ and every real number $\lambda \geqslant 0$.

Suppose we are given a coherent risk measure ρ. Let $\mathcal{A}_\rho = \{x \in E : \rho(x) \leqslant 0\}$. In the mathematical finance literature the previous set is referred to as the set of acceptable positions or the set of "safe" positions. It turns out that $\mathcal{A}_\rho$ is a cone of E which contains P, and for each $x \in E$ we have

$$\rho(x) = \inf\{t \in \mathbb{R} : x + tx_0 \in \mathcal{A}_\rho\}. \tag{1}$$

In section 4 we will take advantage of the following useful result (see Kountzakis and Polyrakis (2013, Theorem 1)):

Lemma 1. *Suppose that E is a Banach space ordered by the cone P. If x_0 is an order unit of E, then the function*

$$\rho(x) = \inf\{t \in \mathbb{R} : x + tx_0 \in P\} \text{ for every } x \in E, \tag{2}$$

is a coherent risk measure with respect to the cone P and the vector x_0. Moreover, if P is closed then $A_\rho = P$.

In what follows we will refer to function (2) as the coherent risk measure defined on E with respect to the cone P and the order unit x_0. In order to state the next result, we need to set up a piece of notation. Let E^* be the topological dual of E, and let P° be the dual cone of P in E^*, i.e.,

$$P^\circ = \{x^* \in E^* : x^*(x) \geqslant 0 \text{ for every } x \in P\}.$$

We denote by B_{x_0} the base for the cone P° which is defined by x_0, i.e.,

$$B_{x_0} = \{x^* \in P^\circ : x^*(x_0) = 1\}.$$

For ease of reference we state below, without proof, Kountzakis and Polyrakis (2013, Theorem 5):

Lemma 2. *If* E *is a Banach space and* ρ *is the risk measure defined on* E *with respect to the closed cone* $P \subseteq E$ *and the order unit* $x_0 \in P$*, then for any* $x \in E$ *we have*

$$\rho(x) = -\min\{x^*(x) : x^* \in B_{x_0}\}. \qquad (*)$$

Definition 1. *(The prudence property). A coherent risk measure* $\rho : E \to \mathbb{R}$ *satisfies the prudence property if* $A_\rho = \{x \in E : \rho(x) \leqslant 0\} = P$.

Remark 1. Observe that not all coherent risk measures defined on a Banach space whose positive cone is closed satisfy the prudence property. To see this, consider the function $\rho : \ell_\infty \to \mathbb{R}$ defined by $\rho(x) = -\liminf_{n\to\infty}\{x_n\}$. It is easy to see that such a function satisfies the four properties listed above (where $x_0 = (1,1,...1,...)$), hence it is a coherent risk measure. Now, let $x = (x_n) = \left\{\left(\frac{(-1)^n}{n}\right) : n \in \mathbb{N}\right\}$ and note that $x \in A_\rho$ however $x \notin \ell_\infty^+$.

The significance of the prudence property can be understood as follows. The set A_ρ collects all acceptable (or "safe") positions with respect to a given coherent risk measure ρ. The idea is that the larger A_ρ, the "less prudent" the corresponding ρ. In fact, if we take two different coherent risk measures, say ρ, ρ', such that $A_\rho \subset A_{\rho'}$, then every x which is acceptable according to ρ is also acceptable according to ρ', but the converse is not true. Clearly, ρ' considers acceptable "actions'"[2] which would be discarded by ρ. This suggests that ρ' is a measure "taking more risk" than ρ, or less stringent than ρ. Following this line of reasoning, we conclude that the most prudent coherent risk measures are those whose corresponding set of acceptable positions A_ρ is the smallest possible, namely P [3]. Therefore, maximizing prudence is in a sense equivalent to minimizing A_ρ as a subset of E.

3 First scenario: the prudent social planner and the Rawlsian function

In what follows it will be useful to think of an infinite sequence of utilities as the outcome of a long-term policy implemented by a social planner (i.e., a policy that affects the welfare of current and future generations). We assume, henceforth, that the set of feasible (infinite) utility streams is

$$X = \ell_\infty = \left\{x = (x_1, x_2, \cdots, x_t, \cdots) \in \mathbb{R}^{\mathbb{N}} : \sup_{t\in\mathbb{N}} |x_t| < \infty\right\},$$

(i.e., the space of bounded real sequences), which is standard in the literature. The generic sequence in ℓ_∞ will be denoted by $x = (x_n)$. We assume, furthermore, that X is ordered by the cone ℓ_∞^+ (the pointwise ordering) and is equipped with the supremum norm. We will let 1 denote the

[2] What an "action" is depends on the context under consideration. In the mathematical finance literature, the notion of coherent risk measure originates in, an action is the random payoff of an asset or portfolio.

[3] Recall that, as noted above, $A_\rho \supseteq P$ for every coherent risk measure ρ.

constant sequence $(1,1,...1,...)$. It is well known that ℓ_∞ endowed with the supremum norm is a Banach space and $1 \in \ell_\infty^+$ is an order unit of ℓ_∞.

Incidentally, let us digress to explain why one would want to choose $1 \in \ell_\infty^+$ as an order unit of ℓ_∞. Suppose that we use $x_0 \neq 1$ instead of 1 as order unit of ℓ_∞. Note that $x_0^n > 0$ for all n. Now, as in the proof of Proposition 1 below one can readily verify that $\inf\{t \in \mathbb{R} : x + tx_0 \in \ell_\infty^+\} = -\inf\left\{\left(\frac{x_n}{x_0^n}\right)\right\}$, where $\inf\left\{\left(\frac{x_n}{x_0^n}\right)\right\}$ can be interpreted as a generalized or weighted Rawlsian social welfare function (the weight assigned to each generation depends on the order unit initially chosen). On the other hand, recall from Lemma 1 that $\inf\{t \in \mathbb{R} : x + tx_0 \in \ell_\infty^+\} = \rho(x)$ is a coherent risk measure (with respect to ℓ_∞^+ and the order unit x_0). The bottom line is that if we used an order unit different from 1 we could simply rephrase Proposition 1 below[4] in terms of a weighted Rawlsian welfare function, exactly like the one mentioned above.

Next, imagine a "risk averse" social planner that is called upon to select infinite utility streams (i.e., policies to implement). Suppose that the social planner 'doesn't take any chances' in fulfilling its task. We argue that such a social planner may want to use a function $W : X \to \mathbb{R}$ which is such that: (i) W is a canonical social welfare function, and (ii) $-W$ is a coherent risk measure. This is because, analogously to the interpretation of the set $\mathcal{A}_\rho = \{x \in E : \rho(x) \leqslant 0\}$ when ρ is a coherent risk measure, by means of this choice the social planner can then make sense of $\mathcal{A}_{-W} = \{x \in X : W(x) \geqslant 0\}$ as the set of socially acceptable or safe utility streams (hence, the set it should make its choice from) which coincides with the set of streams that yield a non-negative social welfare value. Next, we present two examples of social welfare functions W such that $-W$ is a coherent risk measure.

Example 1. Let $\delta \in (0,1)$ and define $W : \ell_\infty \to \mathbb{R}$ by $W(x) := \sum_{n=0}^{\infty} \delta^{n+1} x_n$. This is the standard social welfare function widely used in the macroeconomics literature. We claim that $-W$ is a coherent risk measure. Indeed, since the foregoing discounted sum is a positive linear functional, it is straightforward to check that $-W$ satisfies the defining properties 1, 3 and 4 of coherent risk measures. As for property 2, note that fixing $x_0 := (\frac{1}{\Delta}, \frac{1}{\Delta}, \frac{1}{\Delta}, \cdots, \frac{1}{\Delta}, \cdots)$, with $\Delta := \sum_{n=0}^{\infty} \delta^{n+1}$, we obtain

$$W(x + tx_0) := \sum_{n=0}^{\infty} \delta^{n+1}(x_n + \frac{t}{\Delta}) = \sum_{n=0}^{\infty} \delta^{n+1} x_n + \frac{t}{\Delta} \sum_{n=0}^{\infty} \delta^{n+1} = W(x) + t.$$

Note that if $\delta = \frac{1}{2}$ one obtains $x_0 = 1 := (1,1,1,\ldots)$, which is the assumed order unit of the space of utility streams.

Example 2. Define $W : \ell_\infty \to \mathbb{R}$ by $W(x) := \liminf_{n\to\infty}\{x_n\}$. As explained in the above Remark 1 $-W$ is a coherent risk measure.

In light of the above examples, the planner knows that there are multiple functions it may choose from. The planner realizes that the feasible set it faces, $\mathcal{A}_{-W}$, depends on the choice of W , but it is also aware that $\mathcal{A}_{-W}$ is a cone of ℓ_∞ which contains ℓ_∞^+, no matter the function it chooses. This implies that a utility stream in ℓ_∞^+ is socially 'acceptable' for all welfare functions with the

[4]and the proof of it would remain basically unchanged.

property (ii) mentioned above. Therefore, if we assume that our social planner is 'prudent', i.e., it'd rather not select a utility stream which is socially acceptable for some W but 'unsafe' according to other social welfare functions, then in order 'to be on the safe side' the social planner would restrict attention to only those utility streams that are socially 'acceptable' according to every welfare function that satisfies the aforementioned property (ii). To this end, the planner would have to choose a function satisfying the above properties (i) and (ii) and such that $A_{-W} = \{x \in X : W(x) \geqslant 0\} = \ell_\infty^+$.[5] At this point the planner wonders whether any such a function exists and, if so, what it looks like. Well, Proposition 1 below provides the answer to the foregoing planner's questions and suggests that a social planner with the above features is bound to use the Rawls's social welfare function, whose definition is given below:

Definition 2. *A social welfare function* $W : X \to \mathbb{R}$ *is the Rawlsian function if* $W(x) := \inf_n\{(x_n)\}$ *for all* $x \in \ell_\infty$.

4 The characterization theorem

In this section we prove a characterization theorem which links the notion of coherent risk measure to the Rawls's social welfare function. As we have already outlined above, from a normative point of view the following Proposition 1 formalizes the loose intuition that the Rawls's function is the social welfare function which should be used by a 'prudent' social planner as the criterion that guides the choice of infinite utility streams. As will be clear from the proof of Proposition 1, the Rawlsian social welfare function can be written as the negative of a coherent risk measure (with respect to the cone ℓ_∞^+ and the order unit 1). Therefore, maximizing social welfare requires the social planner to increase as much as possible the utility of the worst-off generation (or member of society), which, in turn, amounts to choosing a sequence of utilities (or an underlying policy generating it) that minimizes the 'riskiness' of utility streams. Thus, the coherent risk measure that characterizes the Rawlsian function could be construed as the risk of becoming the least well-off person in society.

Proposition 1. *Let* $X = \ell_\infty$ *be ordered by the cone* ℓ_∞^+ *and be equipped with the supremum norm. Take* 1 *as an order unit of* ℓ_∞*. Then, a social welfare function* $W : X \to \mathbb{R}$ *is the Rawlsian welfare function if and only if* $-W$ *is a coherent risk measure which satisfies the prudence property.*

Proof. In view of Definition 2, assume that $W(x) := \inf_n\{(x_n)\}$ for all $x \in \ell_\infty$. Next, we claim that W can be written as $-\rho$, where ρ is the function displayed in formula (2). To see this, fix any arbitrary $x = (x_n) \in \ell_\infty$ and note that

[5] Thus, $-W$ is a prudent coherent risk measure (see the above definition of the prudence property). Incidentally, this explains why we have referred to the social planner under consideration as 'prudent'.

$$\begin{aligned}\inf\{t\in\mathbb{R}:x+t1\in\ell_\infty^+\} &= \inf\{t\in\mathbb{R}:x_n+t\geqslant 0 \text{ for every } n\}\\ &= \inf\{t\in\mathbb{R}:t\geqslant -x_n \text{ for every } n\}\\ &= \sup\{(-x_n)\} = -\inf\{(x_n)\}. \end{aligned} \tag{3}$$

Therefore, by Lemma 1 above $-W$ is a coherent risk measure which satisfies the prudence property. For the converse, assume that $W: X\to\mathbb{R}$ is a social welfare function such that $-W$ is a coherent risk measure satisfying the prudence property. Then, by (1) above we have

$$-W(x) = \inf\{t\in\mathbb{R}:x+t1\in\mathcal{A}_{-W}\}, \text{ for every } x\in\ell_\infty. \tag{4}$$

On the other hand, from the prudence property it readily follows that $\mathcal{A}_{-W}=\ell_\infty^+$. So, using (4) we get $-W(x)=\inf\{t\in\mathbb{R}:x+t1\in\ell_\infty^+\}$ for every $x\in\ell_\infty$. The previous equation and the same calculations as those carried out in (3) yield $-W(x)=-\inf\{(x_n)\}$, hence $W(x)=\inf\{(x_n)\}$ for every $x\in\ell_\infty$, as was to be proven. ∎

5 Second scenario: multiple-prior expected utility preferences as the foundation for Rawls's welfare function

In this section we present a formal background to Rawls's thought experiment (the so-called "original position") which lies at the heart of his argument for the principles of social justice he put forward (i.e., promoting the interests of the least advantaged members of society etc.). The background we have in mind can be explained as follows: there are countably-many social states/positions. Consider the generic utility stream $u=(u_n)=(u_1,...u_s,..)$, where u_s is the utility level enjoyed in state/position s. Consider a person who fits the characteristics of the 'rational person' originally envisioned by Rawls, i.e., a person who has no a priori knowledge of her social state in the future, hence she ignores the utility level she will end up with. Also, such a person has too little information to determine the probabilities of the future states (she sits 'behind a veil of ignorance', as Rawls would put it), therefore this person is endowed with a set of probability measures over the states as possible priors. Furthermore, in line with Rawls's ideas, the person at hand is uncertainty-averse, in the sense that she is 'concerned' about the worst-case scenario (i.e., finding herself in the worst state, utility-wise). In view of the above, it may be reasonable to assume that the person under consideration is equipped with preferences represented by a multiple-prior expected utility function.[6]

As we are about to show, it turns out that the Rawlsian social welfare of the given utility stream is equal to the person's multiple-prior expected utility of the given stream. Hence, maximizing

[6]Axiomatic foundations of the multi-prior expected utility representation can be found in Gilboa and Schmeidler (1989).

Rawls's welfare function is equivalent to choosing a utility stream that maximizes the person's preferences. Therefore, we conclude that if a utility-maximizing person, like the one imagined by Rawls, were to choose among all possible welfare functions, it would be rational for her to select the Rawlsian welfare function.

To substantiate the above claims, let's develop a simple and formal treatment of the subject matter that employs basic facts about the sequence space ℓ_∞. For the (omitted) details regarding the following mathematical analysis, we refer the reader to the chapter on the Riesz representation theorems in Aliprantis and Border (2006).

Recall that ℓ_∞ equipped with the supremum norm is a Banach space, ℓ_∞^+ is a closed cone, and $1 \in \ell_\infty^+$ is an order unit of ℓ_∞. Therefore, it follows from (3), (2) in Lemma 1, and $(*)$ in Lemma 2, that for every $u = (u_n) \in \ell_\infty$ we have

$$\inf\{(u_n)\} = \min\{x^*(u) : x^* \in B_1\}, \tag{5}$$

where the sequence u is an infinite stream of contingent utilities (i.e., there is a countable infinity of social states, and the utility level depends on the social state). On the other hand, given any $u = (u_n) \in \ell_\infty$, for each $x^* \in B_1$ there exists a unique finitely additive probability measure μ_{x^*} on $\left(\mathbb{N}, 2^{\mathbb{N}}\right)$, satisfying

$$x^*(u) = \int_{\mathbb{N}} u \, d\mu_{x^*}. \tag{6}$$

To see an explicit example and fix ideas, consider the functional $x^* : \ell_\infty \to \mathbb{R}$ defined by $x^*(u) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n u_n$ for all $u = (u_n) \in \ell_\infty$, and note that $x^* \in B_1$. The associated $\mu_{x^*} : 2^{\mathbb{N}} \to [0,1]$ is then given by

$$\mu_{x^*}(A) := \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \cdot \chi_A(n),$$

where $\chi_A : \mathbb{N} \to \{0,1\}$ is the characteristic function of A. Hence, it should be clear that the above functional can be written as follows:

$$x^*(u) = \int_{\mathbb{N}} u \, d\mu_{x^*}.$$

Thus, we can view $x^*(u)$ as the expected utility $\mathbb{E}_{\mu_{x^*}}[u]$; indeed, if we think of u as a list of possible outcomes u_n for each state $n \in \mathbb{N}$, the above discounted sum $x^*(u) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n u_n$ is exactly the sum of the outcomes weighted by the probability that the respective state $n \in \mathbb{N}$ occurs.

Now, imagine a social planner that sets ex-ante a utility stream u, and consider a person (described above) who is uncertain as to what social position she will occupy in the future. Suppose such person's beliefs about social states are given by μ_{x^*}. Then, in light of (6) and the above example, $x^*(u)$ may be thought of as the aforementioned person's expected utility, and will be denoted by $\mathbb{E}_{\mu_{x^*}}[u]$. Therefore, given any $u = (u_n) \in \ell_\infty$, (5) above can be rearranged as

$$\inf\{(u_n)\} = \min\left\{\mathbb{E}_{\mu_{x^*}}[u] : x^* \in B_1\right\} = \min\{\mathbb{E}_\pi[u] : \pi \in \Delta\}, \tag{7}$$

where $\Delta = \{\mu_{x^*} : x^* \in B_1\}$. Note that the term on the left hand-side member of (7) is the Rawlsian social welfare of the given utility stream, and the second term on the right hand-side member is the person's multiple-prior expected utility evaluated at u, where the set (of priors) Δ captures this person's ambiguous beliefs about the states. Therefore, we come to the conclusion that if the person at hand were a selfish utility-maximizer, it would be in her best interest to propose using the Rawls's social welfare function.

6 Concluding remarks

In the mainstream social choice and welfare literature it is customary to characterize various social welfare functions in terms of ethical principles that are set out as mathematical axioms. However, in this work we have taken a different route. We have borrowed the concept of coherent risk measure from mathematical finance and we have exploited it to characterize the Rawls's social welfare function. To the best of our knowledge, this is a novel result. As we have explained above, our characterization result allows us to achieve two objectives: we identify a setting where it is logical for a social planner to choose the Rawlsian welfare function over other feasible social welfare criteria, and we are also able to formalize the thought experiment which was conceived of by Rawls to advocate for his maxmin criterion as the appropriate way to measure social welfare.

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